Some Mathematical Models to Turn Social Media into Knowledge

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Some Mathematical Models

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Pinding Chatty Users (Multi-Armed Bandit)

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Outline



2 Finding Chatty Users (Multi-Armed Bandit)

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Spatio-temporal Signal: When, Where, How Much





"100 dead robins found in New York last Friday"

Transportation Safety



"16 deer got run over by cars in **Wisconsin last month** "

• Direct instrumental sensing is difficult and expensive

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Social Media Users as Sensors



- Not "hot trend" discovery: We know what event we want to monitor
- We are given a reliable text classifier for "hit"
- Our task: precisely estimating a spatiotemporal intensity function f_{st} of a pre-defined target phenomenon.

Challenges of Using Humans as Sensors

- Keyword doesn't always mean event
 - I was just told I look like dead crow.
 - Don't blame me if one day I treat you like a dead crow.
- Human sensors aren't under our control
- Location stamps may be erroneous or missing, e.g., in Twitter
 - ▶ 3% have GPS coordinates: (-98.24, 23.22)
 - ▶ 47% have valid user profile location: "Bristol, UK, New York"
 - 50% don't have valid location information
 "Hogwarts, In the traffic..blah, Sitting On A Taco"

Problem Definition

- Input: A list of time and location stamps of the target posts.
- Output: f_{st} Intensity of target phenomenon at location s (e.g., New York) and time t (e.g., 0-1am)

Time	Location			Time (<i>t</i>)		
2012-09-26 17:35:23	New York US			0-1am	1-2am	2-3am
2012-09-27	N/A	(<i>s</i>)	California	f(1,1)	f(1,2)	f(1,3)
12:17:52	ation	New York	f(2,1)	f(2,2)	f(2,3)	
2012-09-27 08:28:12	(-98.24, 23.22)	Loca	Washington	f(3,1)	f(3,2)	f(3,3)

Why Simple Estimation is Bad

- $f_{st} = x_{st}$, the count of target posts in bin (s,t)
- Justification: MLE of the model $x \sim \text{Poisson}(f)$

$$P(x) = \frac{f^x e^{-f}}{x!}$$

However,

- \blacktriangleright Population Bias: Assume $f_{st} = f_{s't'}$, if more users in (s,t) , then $x_{st} > x_{s't'}$
- Imprecise location: Posts without location stamp, noisy user profile location
- Zero/Low counts: If we don't see tweets from Antarctica, no penguins there?

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Correcting Population Bias

• Social media user activity intensity g_{st}

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x \sim \text{Poisson}(\eta(f,g))
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- Link function (target post intensity) $\eta(f,g) = f \cdot g$
- Count of all posts $z \sim \text{Poisson}(g)$
- g_{st} can be accurately recovered



Handling Imprecise Location





[Reproduced from Vardi et al(1985), A statistical model for positron emission tomography]

Handling Imprecise Location: Transition



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Handling Imprecise Location: Transition



location stamps

 $x_i \sim Poisson(h_i)$

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Handling Zero / Low Counts



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The Graphical Model



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Optimization and Parameter Tuning

$$\min_{\theta \in \mathbb{R}^n} - \sum_{i=1}^m \left(x_i \log h_i - h_i \right) + \lambda \Omega(\theta)$$
$$\theta_j = \log f_j$$
$$h_i = \sum_{j=1}^n P_{ij} \eta(\theta_j, \psi_j)$$

- Quasi-Newton method (BFGS)
- Cross-Validation: Data-based and objective approach to regularization; Sub-sample events from the total observations

Theoretical Consideration

- How many posts do we need to obtain reliable recovery?
- If $x \sim \text{Poisson}(h)$, then $\mathbb{E}[(\frac{x-h}{h})^2] = h^{-1} \approx x^{-1}$: more counts, less error
- Theorem: Let f be a Hölder α -smooth d-dimensional intensity function and suppose we observe N events from the distribution Poisson(f). Then there exists a constant $C_{\alpha} > 0$ such that

$$\inf_{\widehat{f}} \sup_{f} \frac{\mathbb{E}[\|\widehat{f} - f\|_{1}^{2}]}{\|f\|_{1}^{2}} \geq C_{\alpha} N^{\frac{-2\alpha}{2\alpha+d}}$$

 \bullet Best achievable recovery error is inversely proportional to N with exponent depending on the underlying smoothness

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Roadkill Spatio-Temporal Intensity

- The intensity of roadkill events within the continental US
- Spatio-Temporal resolution: State: 48 continental US states, hour-of-day: 24 hours
- Data source: Twitter
- Text classifier: Trained with 1450 labeled tweets. CV accuracy 90



Text preprocessing

- Twitter streaming API: animal name + "ran over"
- Remove false positives by text classification
 "I almost ran over an armadillo on my longboard, luckily my cat-like reflexes saved me."
- Feature representation
 - Case folding, no stemming, keep stopwords
 - ▶ @john \rightarrow @USERNAME, http://wisc.edu \rightarrow HTTPLINK, keep #hashtags, keep emoticons
 - Unigrams + bigrams
- Linear SVM
 - Trained on 1450 labeled tweets outside study period
 - Cross validation accuracy 90%

Chipmunk Roadkill Results



Roadkill Results on Other Species



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Outline

Spatio-Temporal Signal Recovery (Poisson Generative Model)

Pinding Chatty Users (Multi-Armed Bandit)

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Finding Chatty Users

- Find top k social media users on a topic
 - ► For example, via the bullying classifier [Xu,Jun,Zhu,Bellmore NAACL 2012]
- Trivial if we can monitor all users all the time
- But API only allows monitoring a small number (e.g. 5000) of users at a time
- Monitor each user "long enough?"

How Long is Long Enough for a Single User?

- Define a time slot (e.g., 1 hour)
- Define Boolean event
 - ▶ 1= the user posted anything on-topic in the time slot
 - 0= no post

• Define
$$p = Pr(event=1)$$

• Observe k time slots $X_1, \ldots, X_k \in \{0, 1\}$

•
$$\hat{p} = \frac{\sum_{i=1}^{k} X_i}{k}$$

• How reliable is \hat{p} ?

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Hoeffding's Inequality

Let X_1, \ldots, X_k be independent with $P(X_i \in [a, b]) = 1$ and the same mean p. Then for all $\epsilon > 0$,

$$P\left(\left|\frac{1}{k}\sum_{i=1}^{k}X_{i}-p\right| > \epsilon\right) \le 2e^{-\frac{2k\epsilon^{2}}{(b-a)^{2}}}.$$

• We have
$$P\left(\left| \hat{p} - p \right| > \epsilon
ight) \leq 2e^{-2k\epsilon^2}$$

• Define
$$\delta = 2e^{-2k\epsilon^2}$$
, then $\epsilon = \sqrt{\frac{\log \frac{2}{\delta}}{2k}}$ or $k = \frac{\log \frac{2}{\delta}}{2\epsilon^2}$.

• For any $\delta > 0$, with probability at least $1 - \delta$, $|\hat{p} - p| \le \sqrt{\frac{\log \frac{2}{\delta}}{2k}}$.

- With $\frac{\log \frac{2}{\delta}}{2\epsilon^2}$ samples, with probability at least 1δ , $|\hat{p} p| \le \epsilon$.
- Confidence interval or Probably-Approximately-Correct (PAC) analysis

Uniform Monitoring is Wasteful

To find an ϵ -best arm $(p > p_1 - \epsilon)$ out of n arms, uniform monitoring needs a total of $O\left(\frac{n}{\epsilon^2}\log\frac{n}{\delta}\right)$ samples [Even-Dar et al. 2006]



Median Elimination (a Multi-Armed Bandit algorithm) needs $O\left(\frac{n}{\epsilon^2}\log\frac{1}{\delta}\right)$ samples



One-Armed Bandit



Expected reward $p \in [0, 1]$

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Stochastic Multi-Armed Bandit Problem

- Known parameters: number of arms n.
- Unknown parameters: n expected rewards $p_1 \ge \ldots \ge p_n \in [0, 1]$.
- For each round $t = 1, 2, \ldots$
 - **1** the learner chooses an arm $a_t \in \{1, \ldots, n\}$ to pull
 - **2** the world draws the reward $X_t \sim \text{Bernoulli}(p_{a_t})$ independent of history
- The learner does not see the reward of non-chosen arms in each round.
- Pure exploration problem: find arms with the largest *p*'s as quickly as possible.

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Chatty Users as Multi-Armed Bandit

- User = arm, n = number of users (e.g. millions)
- $p_i = \text{user } i$'s probability to post
- Monitoring user for a time slot = pulling that arm
- Reward $X_t = did$ the user post anything?
- Pure exploration: with the least monitoring, find top-m users who post the most $(m \ll n)$
 - exactly the top-m users $p_1 \ge \ldots \ge p_m$, or
 - approximately the top-m users?

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Approximate Top-*m* Arms: Strong vs. Weak Guarantee

- Let $S \subseteq \{1, \ldots, n\}$ and let S(j) denote the arm whose expected reward is *j*-th largest in *S*.
- S is strong (ϵ, m) -optimal if $p_{S(i)} \ge p_i \epsilon, \forall i = 1, ..., m$.
- S is weak (ϵ, m) -optimal if $p_{S(i)} \ge p_m \epsilon, \forall i = 1, ..., m$.



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Multi-Armed Bandit Algorithms for Finding Top-m Arms

Guarantee	Algorithm	Sample Complexity (Worst Case)
Exact	SAR	$O(\frac{n}{\epsilon^2} \left(\log n\right) \left(\log \frac{n}{\delta}\right))$
Strong (ϵ, m) -optimal	EH	$O(\frac{n}{\epsilon^2}\log\left(\frac{m}{\delta}\right))$
Weak (ϵ, m) -optimal	Halving	$O(\frac{n}{\epsilon^2}\log\left(\frac{m}{\delta}\right))$
Weak (ϵ, m) -optimal	LUCB	$O(rac{n}{\epsilon^2}\log\left(rac{n}{\delta} ight))$

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The Enhanced Halving (EH) Algorithm

1: Input:
$$n, m, \epsilon > 0, \delta > 0$$

2: Output: m arms satisfying strong (ϵ, m) -optimality
3: $l \leftarrow 1, S_1 \leftarrow \{1, ..., n\}, n_1 \leftarrow n, \epsilon_1 \leftarrow \epsilon/4, \delta_1 \leftarrow \delta/2$
4: while $n_l > m$ do
5: $n_{l+1} \leftarrow \begin{cases} \lceil n_l/2 \rceil & \text{if } |S_l| > 5m \\ m & \text{otherwise} \end{cases}$
6: Pull every arm in $S_l \left[\frac{1}{(\epsilon_l/2)^2} \log \left(\frac{5m}{\delta_l} \right) \right]$ times

- 7: Compute $\hat{p}_a^{(l)}, a \in S_l$, the empirical means from the sample drawn at iteration l
- 8: $S_{l+1} \leftarrow \{n_{l+1} \text{ arms with largest empirical means from } S_l\}$

9:
$$\epsilon_{l+1} \leftarrow \frac{3}{4} \epsilon_l, \delta_{l+1} \leftarrow \frac{1}{2} \delta_l, \ l \leftarrow l+1$$

- 10: end while
- 11: Output $S := S_l$

Further improvement in constant: the Quantiling algorithm.

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Application to Twitter Bullying

- n = 522,062 users, top m = 100
- 1 month total monitoring time (January 2013)
- $T = 31 \times 24 = 744$ time slots (pulls)
- Batch pulling 5000 arms at a time (user streaming API)
- Reward $X_{it} = 1$ if user *i* posts bullying-related tweets (judged by a text classifier) in time slot *t*.
- log log plot of expected reward follows the power law:



Experiments

Strong error of a set of m arms S:

$$\max_{i=1...m} \{ p_i - p_{S(i)} \}$$

the smallest ϵ with which S is strong (ϵ, m) -optimal.

Methods	Strong Error		
EH	0.1040 (± 0.004)		
Quantiling	0.0478 (± 0.002)		
Halving	0.0999 (± 0.004)		
LUCB	$0.1474 \ (\pm \ 0.004)$		
LUCB/Batch	0.0826 (± 0.004)		
SAR	0.0678 (± 0.003)		
Uniform	0.0870 (± 0.003)		

Summary

- We present two social media mining tasks:
 - estimating intensity from counts
 - identifying the most chatty users
- Naive heuristic methods do not take full advantage of the data
- Mathematical models with provable properties extract knowledge from social media better.
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